Mathematics HS

Mathematics is used as a means to communicate about quantities, logical relationships, and unknowns. Such a simplistic statement may make students who are not planning to go to college ask why mathematics is necessary for them. While the ability to do computation is important, it is the skills of problem finding and problem solving along with developing abstract thinking, symbolic representation and interpretation, logical arguments, and objective reasoning that allow us to function effectively and understand our world.

Mathematics is one area of coursework in the high school curriculum where students are taught these skills and where answers cannot be obtained just by common sense and guessing. Even without an ever-increasing reliance on technology, mathematical skills meet needs for practical everyday life, intelligent citizenship, and future employment. A study by Arizona State University indicated that students who opt out of advanced levels of mathematics and science may now eliminate up to 75% of career opportunities from which to choose[†]. Algebra has been called the academic passport for passage into virtually every facet of the job market. Employers want their employees to be able to set up problems, estimate solutions, identify how accurate solutions need to be, work with other people to reach goals, know the many different types of mathematics that exist, and determine which one is needed in a particular situation. It is clear that the mathematical literacy of the twentieth century will **not** be sufficient for the twenty-first century.

[†]*ASU Research* Fall 1998, p. 41

ABOUT THE TEST

The mathematics test contains approximately 100 multiple-choice questions. Calculators are not allowed; however, the calculations required can be readily handled. The questions will emphasize conceptual understanding, process, and problem solving skills rather than just computation skills.

Hints for Taking AIMS HS – Mathematics

- Remember, this is not a timed test. Take your time and do your best work.
- Since calculators are not allowed on this test, double-check your work!
- Check to see if your answer is reasonable.

Sample Items for Mathematics

What To Expect From This Section

This AIMS HS Student Guide for Mathematics provides examples of the format and types of items that will appear on AIMS HS Mathematics. An attempt has been made to provide a sampling of the types of questions that might be asked; however, not every concept in each strand has a corresponding sample item in this guide. An answer key for all mathematics sample items is provided in the appendices. Additionally, you will find an AIMS HS Mathematics Reference Sheet in the appendices.

Strand 1: Number Sense and Operations

General concepts you should know:

- Real number system and its various subsystems (natural, whole, integers, rationals, and irrationals).
- Operations with positive and negative numbers.
- Scientific notation.
- Estimation strategies.

1 Which of the following sets of numbers is not infinite?

A {natural numbers less than 8}

B {odd integers less than 8}

C {rational numbers less than 8}

D {real numbers less than 8}

2 What is the value of the expression below?

$$27 - (9 - 6)^2 \times 3$$

A 54

B 9

C 0

D -108

3 Eight friends went out to dinner together before prom. The restaurant adds a gratuity (tip) of 15% to the total for groups of 8 or more. The cost of the meals was $270.40, including tax. Which amount is closest to the total cost of dinner, including the gratuity?

A $325

B $311

C $284

D $41

Strand 2: Data Analysis, Probability, and Discrete Mathematics

General concepts you should know:

- Graphs (histograms, line graphs, circle graphs, box-and-whisker plots, frequency charts, stem-and-leaf plots, and scatter plots).
- Measures of central tendency, variability and correlation (mean, median, mode, quartiles, and range).
- Sample vs. census.
- Biased vs. unbiased samples.
- Pattern prediction.
- Misuses of statistics.
- Probability.
- Positive and negative correlation.
- Probable outcomes of events.
- Systematic listing and counting; outcomes sets.
- Use of combinations vs. permutations.

4 The state of Arizona encompasses 113,642 sq. mi. of land and 364 sq. mi. of water. California encompasses 155,973 sq. mi. of land and 7,734 sq. mi. of water. Which matrix below represents these data?

A
$$\begin{array}{c} \\ \text{Arizona} \\ \text{California} \end{array}\begin{array}{c} \begin{array}{cc} \text{Land} & \text{Water} \end{array} \\ \begin{pmatrix} 7,734 & 155,973 \\ 364 & 113,642 \end{pmatrix} \end{array}$$

B
$$\begin{array}{c} \\ \text{Arizona} \\ \text{California} \end{array}\begin{array}{c} \begin{array}{cc} \text{Land} & \text{Water} \end{array} \\ \begin{pmatrix} 155,973 & 7,734 \\ 113,642 & 364 \end{pmatrix} \end{array}$$

C
$$\begin{array}{c} \\ \text{Arizona} \\ \text{California} \end{array}\begin{array}{c} \begin{array}{cc} \text{Land} & \text{Water} \end{array} \\ \begin{pmatrix} 113,642 & 364 \\ 155,973 & 7,734 \end{pmatrix} \end{array}$$

D
$$\begin{array}{c} \\ \text{Arizona} \\ \text{California} \end{array}\begin{array}{c} \begin{array}{cc} \text{Land} & \text{Water} \end{array} \\ \begin{pmatrix} 113,642 & 7,734 \\ 155,973 & 364 \end{pmatrix} \end{array}$$

5 The table below shows the median commuting distance to work (one-way) for given Arizona cities.

City	Median Commute (in miles)
Phoenix	8
Peoria	21
Chandler	15
Mesa	12

Based on the table, which of the following statements must be true?

A People from Mesa have the longest commute.

B All people from Phoenix commute at least 8 miles.

C More people commute from Peoria to Phoenix than from Phoenix to Peoria.

D At least half the population of Chandler has a commute of 15 or fewer miles.

6 Each of the events below is performed randomly. Which includes a dependent event?

A A card is drawn from a deck of playing cards, replaced in the deck, and a second card is drawn from the deck.

B A spinner with 8 congruent sectors is spun, the number is marked, and the spinner is spun again.

C A fair coin is flipped, the side it landed on is marked, and the coin is flipped again.

D Twenty differently numbered tiles are put into a bag. One tile is drawn, the number is marked, the tile set aside, and a second tile is drawn.

7 The principle wants to read the list of candidates for prom queen. There are 6 candidates. How many ways can the principle introduce the candidates?

A 2160

B 720

C 21

D 6

Strand 3: Patterns, Algebra, and Functions

General concepts you should know:

- Graphing, evaluating, simplifying, and solving linear equations and inequalities.
- Real and rational roots.
- Systems of linear equations.
- Domain and range equations.
- Graphical representations of function ("vertical line" test).

8 Mrs. Herrera started a sequence of numbers by adding the first three terms to get the fourth term. Her first three terms were 3, 5, and 8, which gave her 16 as the fourth term. To get each new term she added the three preceding terms. The first 5 terms of her sequence are below.

3, 5, 8, 16, 29, …

Which of the following is the 7^{th} term in her sequences?

A 45

B 53

C 98

D 106

9 Jamal calculated that it costs \$0.43 per mile and \$4.00 per day to operate his car. If Jamal drive m miles over d days, which equation below expresses the cost, C, of driving the car, in terms of m and d?

A $C = \$4.00d + \$0.43m$

B $C = \$0.43d + \$4.00m$

C $C = \$4.30d + \$4.00m$

D $C = \$4.00d + \$4.30m$

10 Which value of y makes the proportion below true?

$$\frac{5}{-3+y} = \frac{8}{y+6}$$

A $y = 2$

B $y = 3$

C $y = 11$

D $y = 18$

11 What is the sum of the solutions for the quadratic equation below?

$$3x^2 + x - 2 = 0$$

A $-1\frac{2}{3}$

B $-\frac{1}{3}$

C $\frac{1}{3}$

D $1\frac{2}{3}$

12 Given: $A = \begin{pmatrix} -5 & 3 \\ 4 & -3 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ 0 & 7 \end{pmatrix}$

Which of the following is the sum of $2A + 3B$?

A $\begin{pmatrix} -4 & 3 \\ 8 & 15 \end{pmatrix}$

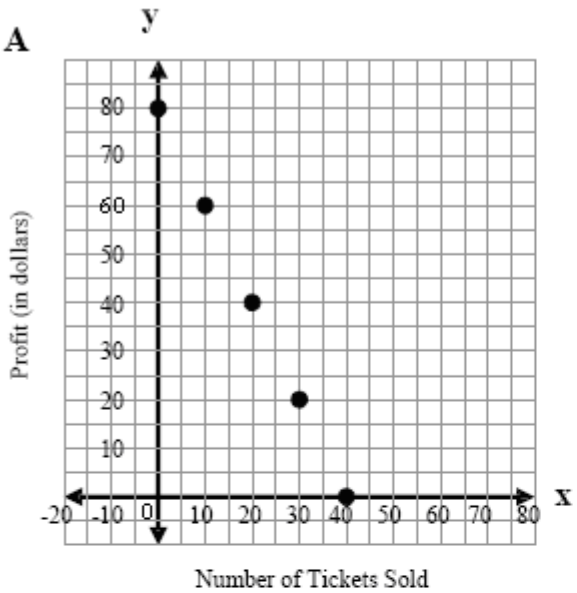
B $\begin{pmatrix} -16 & 9 \\ 8 & -27 \end{pmatrix}$

C $\begin{pmatrix} -16 & 9 \\ 12 & 5 \end{pmatrix}$

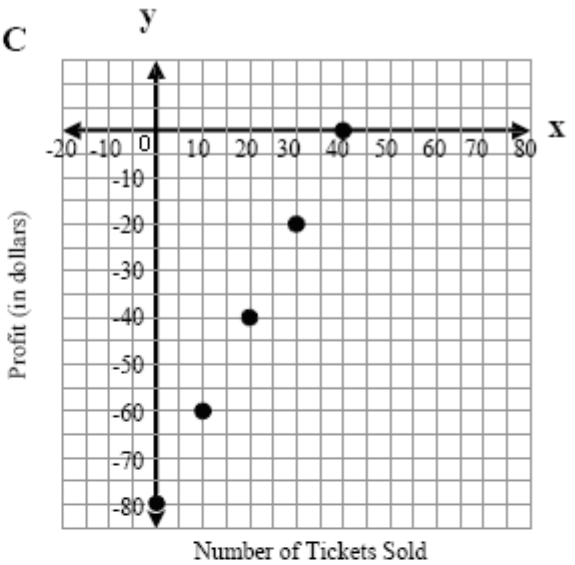
D $\begin{pmatrix} -4 & 3 \\ 4 & 4 \end{pmatrix}$

13 The student council at a school held a raffle using donated prizes. They spent $80 for promotions and additional prizes. The tickets were sold for $2 each. Which graph best represents their total profits as a function of the number of tickets sold?

A



C



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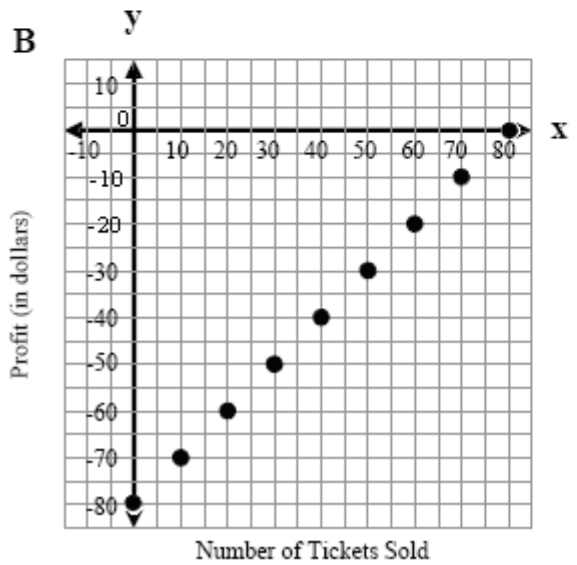

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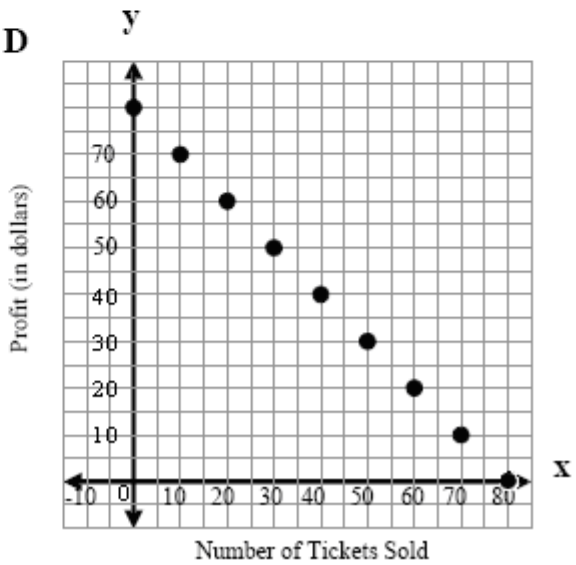
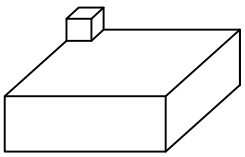


Strand 4: Geometry and Measurement

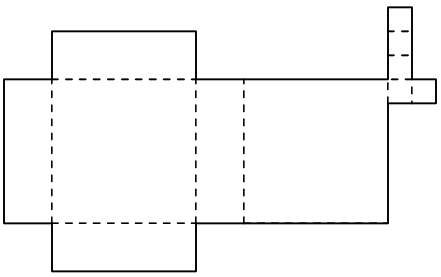
General concepts you should know:

- Pythagorean theorem.
- Triangle characteristics (congruence and similarity relationships).
- Angle characteristics (complementary, supplementary, and congruent).
- Circle characteristics (arcs, chords, tangents, and secants).
- Identification of prisms, pyramids, cones, cylinders, and spheres.
- Coordinate plane characteristics (coordinates, distance and midpoint).
- Transformations (reflections, rotations, dilations, translations; symmetry).
- Appropriate units of measure, applications of techniques and formulas.
- Perimeter, area, volume; measuring line segments, lines, angles, 2-D and 3-D figures.

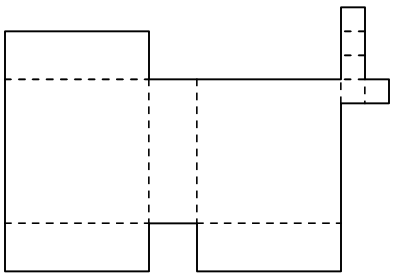
14 Which net below best represents the following object?



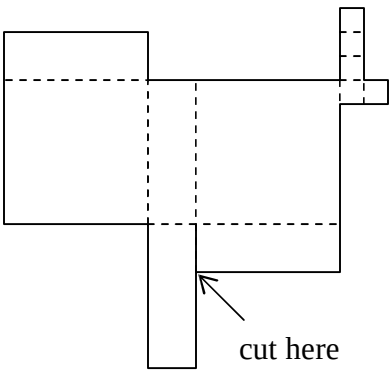
A



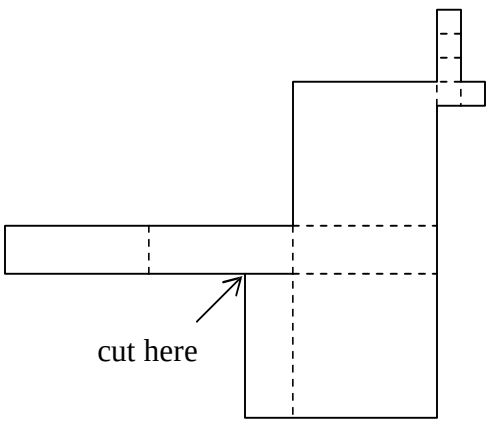
C



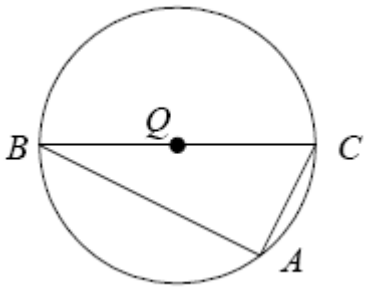
B



D



15 The points *A*, *B*, and *C* lie on circle *Q* below, in which $\overline{BC}$ is a diameter.



In circle *Q*, what is the measure of angle *CAB*, in degrees?

A 360˚

B 180˚

C 90˚

D 60˚

16 What is the length of the line segment that has endpoints at (-5, 3) and (4, 5)?

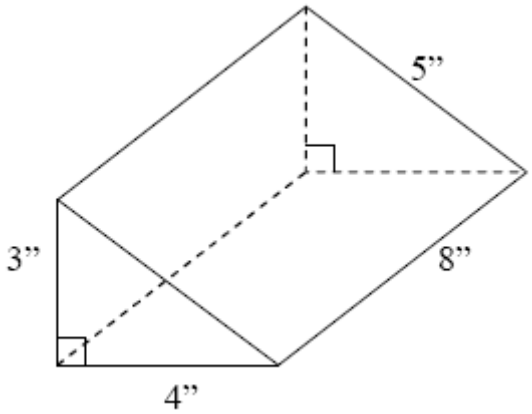
A $\sqrt{121}$

B $\sqrt{85}$

C $\sqrt{65}$

D $\sqrt{11}$

17 What is the surface area of the triangular prism represented below in square inches?



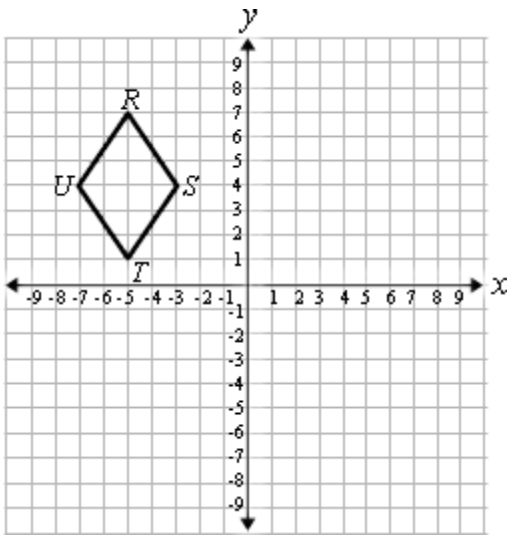
A 102 sq. in.

B 108 sq. in.

C 96 sq. in.

D 48 sq. in.

18 What is the image of *R* when *RSTU* is translated 3 units down and 6 units right?



A (-1, 1)

B (1, -1)

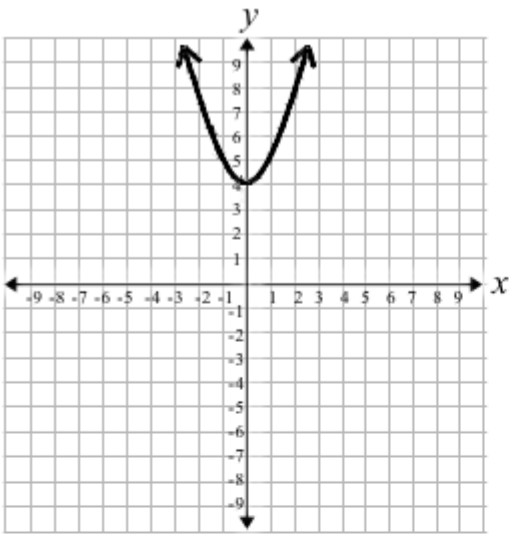
C (1, 4)

D (4, 1)

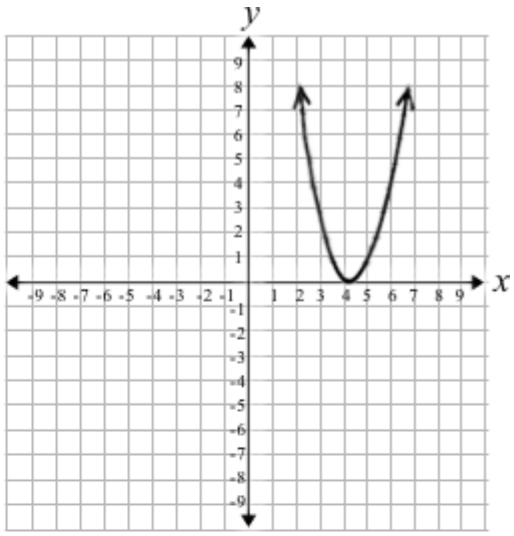
19 Which of the following represents the graph of the equation below?

$$y = x^2 + 4$$

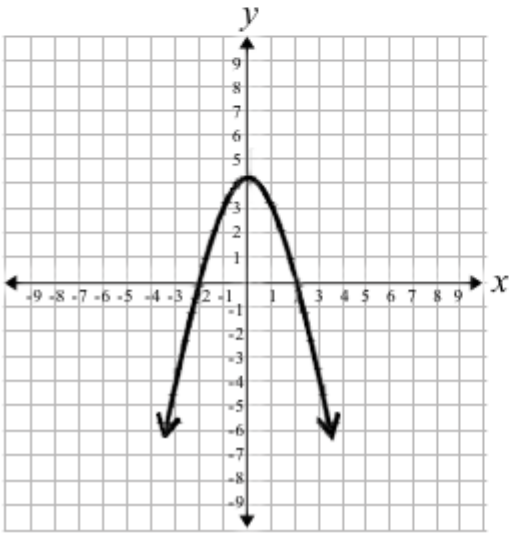
A



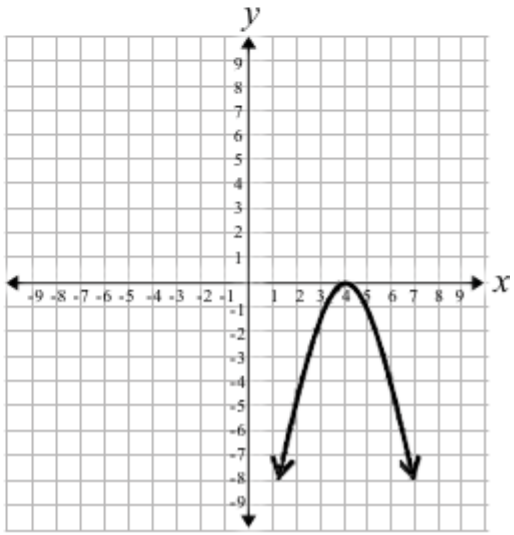
C



B



D



Strand 5: Structure and Logic

General concepts you should know:

- Inductive and deductive reasoning.
- Conjectures.
- Validity of arguments.

20 Which of the following is a correct procedure for solving the equation below?

$2(x - 6) - 12 = -3(x + 5)$

A $2(x - 6) - 12 = -3(x + 5)$
$2x - 6 - 12 = -3x + 5$
$2x - 18 = -3x + 5$
$5x - 18 = 5$
$5x = 23$
$x = \frac{23}{5}$

B $2(x - 6) - 12 = -3(x + 5)$
$2x - 12 - 12 = -3x + 15$
$2x = -3x + 15$
$5x = 15$
$x = 3$

C $2(x - 6) - 12 = -3(x + 5)$
$2x - 12 - 12 = -3x - 15$
$2x - 24 = -3x - 15$
$5x - 24 = -15$
$5x = -39$
$x = \frac{39}{5}$

D $2(x - 6) - 12 = -3(x + 5)$
$2x - 12 - 12 = -3x - 15$
$2x - 24 = -3x - 15$
$5x - 24 = -15$
$5x = 9$
$x = \frac{9}{5}$

21 Let n be any irrational number. Which of the following is always true about $\frac{n}{2}$?

A $\frac{n}{2}$ is a whole number

B $\frac{n}{2}$ is an odd integer

C $\frac{n}{2}$ is a prime number

D $\frac{n}{2}$ is an irrational number

AIMS Reference Sheet

Formulas for Area

Triangle	$A = \frac{1}{2}bh$
Rectangle	$A = lw$
Trapezoid	$A = \frac{1}{2}h(b_1 + b_2)$
Parallelogram	$A = bh$
Circle	$A = \pi r^2$

Key

b = base	d = diameter
h = height	r = radius
l = length	ℓ = slant height
w = width	B = area of base
	P = perimeter of base

Use 3.14 or $\frac{22}{7}$ for π.

Formulas for Volume and Area of Solids

Solid	Volume	Total Surface Area
Right Circular Cone	$V = \frac{1}{3}\pi r^2 h$	$T = \frac{1}{2}(2\pi r)\ell + \pi r^2 = \pi r \ell + \pi r^2$
Pyramid	$V = \frac{1}{3}Bh$	$T = B + \frac{1}{2}P\ell$
Sphere	$V = \frac{4}{3}\pi r^3$	$T = 4\pi r^2$
Right Circular Cylinder	$V = \pi r^2 h$	$T = 2\pi rh + 2\pi r^2$
Right Prism	$V = Bh$	$T = 2B + Ph$

Linear Equation Forms

Point–Slope Form:

$$y - y_1 = m(x - x_1)$$

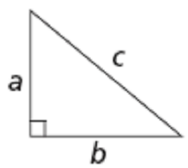
Standard or General Form:

$$Ax + By = C$$

Slope–Intercept Form:

$$y = mx + b$$

Pythagorean Theorem



$$c^2 = a^2 + b^2$$

Coordinate Geometry

Given: Points $A(x_1, y_1)$, $B(x_2, y_2)$

Distance between two points:

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint between two points:

$$\text{Midpoint of } \overline{AB} = \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$$

Slope of line through two points:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Quadratic Formula

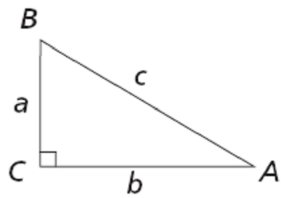
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

AIMS Reference Sheet

Sum of the measures of the interior angles of a convex polygon with n sides: $S = (n - 2)(180°)$	Distance, rate, time formula, where d = distance, r = rate, t = time: $d = rt$
Permutations of n objects taken r at a time: ${}_nP_r = \frac{n!}{(n-r)!}$	Combinations of n objects taken r at a time: ${}_nC_r = \frac{n!}{(n-r)! \cdot r!}$

Right-Triangle Relationships

Trigonometric Ratios

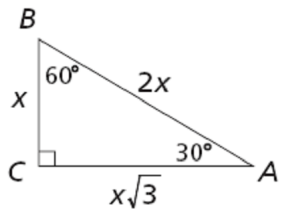


$\sin A = \frac{a}{c}$

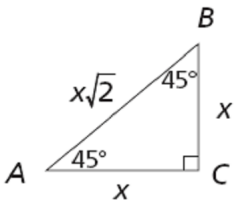
$\cos A = \frac{b}{c}$

$\tan A = \frac{a}{b}$

30°–60°–90° Triangle Relationships



45°–45°–90° Triangle Relationships



Additional Formulas

Circumference $= \pi d = 2\pi r$

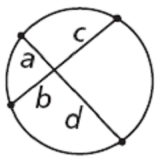
Use 3.14 or $\frac{22}{7}$ for π.

Area of a sector:

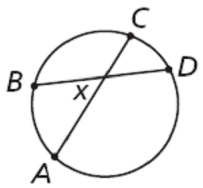
$$A = \pi r^2 \left(\frac{\text{degrees in corresponding arc}}{360°} \right)$$

Length of a circular arc:

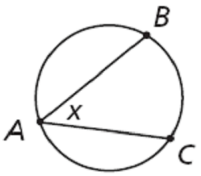
$$\text{Length of } \overset{\frown}{AB} = 2\pi r \frac{m\overset{\frown}{AB}}{360°}$$



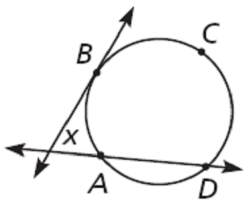
$\frac{a}{b} = \frac{c}{d}$



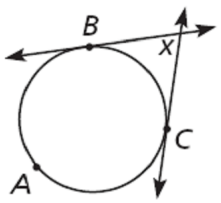
$m\angle x = \frac{1}{2}\left(m\overset{\frown}{AB} + m\overset{\frown}{CD}\right)$



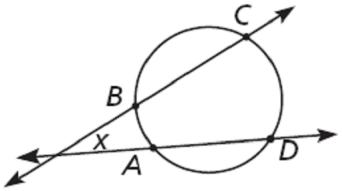
$m\angle x = \frac{1}{2} m\overset{\frown}{BC}$



$m\angle x = \frac{1}{2}\left(m\overset{\frown}{BCD} - m\overset{\frown}{AB}\right)$



$m\angle x = \frac{1}{2}\left(m\overset{\frown}{BAC} - m\overset{\frown}{BC}\right)$



$m\angle x = \frac{1}{2}\left(m\overset{\frown}{CD} - m\overset{\frown}{AB}\right)$

Scoring Key

Mathematics Key

Question #1: A
Question #2: C
Question #3: B
Question #4: C
Question #5: D
Question #6: D
Question #7: B
Question #8: C
Question #9: A
Question #10: D
Question #11: B
Question #12: A
Question #13: C
Question #14: A
Question #15: C
Question #16: B
Question #17: B
Question #18: C
Question #19: A
Question #20: D
Question #21: D